

# COHERENT ELECTROMAGNETIC PROCESSES IN RELATIVISTIC HEAVY ION COLLISIONS

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Using the strong electromagnetic fields in peripheral heavy ion collisions gives rise to a number of interesting possibilities of applications in both photon-photon and photon-hadron physics. We look at the theoretical foundations of the equivalent photon approximation and the specific problems in the heavy ion case. The interesting physics processes that can be studied in this way are outlined. Electron positron pair production plays a special role. We look at multiple pair production and Coulomb corrections as typical strong field effects. But electron positron pair production is also an important loss process and has some practical applications.

## 1 Introduction

Central collisions are the reason, why heavy ion colliders have been or are currently built. But as soon as these colliders are available it is of interest to look also at very peripheral (“ultra-peripheral”) collisions, which are characterized by the condition that the impact parameter  $b$  is larger than the sum of the nuclear radii. The two ions do not interact directly via the nuclear interaction in this case and only the electromagnetic field or the long range part of the nuclear interaction (described by Pomeron- or meson-exchange) is still able to interact.

Peripheral collisions are quite complementary to the central ones, both in terms of the characteristics of the events, but also concerning the physics of interest. Peripheral collisions can be characterized as being “*silent*” compared to the “*violent*” central interaction with their large multiplicities. Whereas in central collisions one is looking for “*matter under extreme conditions*” the focus of peripheral collisions is more towards the “*elementary interaction of photons*”. Peripheral collisions can in many cases study properties which cannot be easily extracted from the central collisions.

The coherent action of all the protons within the nucleus gives an enhancement factor of  $Z^2$  or  $Z^4$ , which is the reason for the often very large cross sections and rates. The strong electromagnetic fields have already found a number of applications in atomic physics and in nuclear physics. In the past years their use at higher energies for both photon-photon and photon-hadron physics has been studied theoretically. Some reviews of the field can be found in <sup>1,2,3</sup>, the most recent one is <sup>4</sup>. With the recent measurements at the STAR detector at RHIC this field has now left the purely theoretical stage and has come into reality.

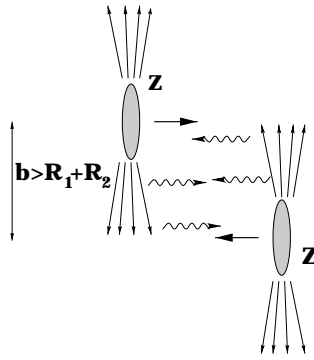


Figure 1: The strong electromagnetic fields surrounding the heavy ions in relativistic collisions are an intense source of (quasireal) photons. They can be used in very peripheral collisions for photon-photon and photon-nucleus collisions.

## 2 The Equivalent Photon Approximation in the heavy ion case

Lepton (electron) colliders have been traditionally used to study  $\gamma\gamma$  physics. The equivalent photon approximation is one of the main theoretical tools there. This method was originally developed by Fermi<sup>5</sup> from the observation that the Coulomb field of a fast moving object resembles the flux of a spectrum of real photons and later extended to the relativistic case by Weizsäcker<sup>6</sup> and Williams<sup>7</sup>. Its application for photon-photon physics is discussed in detail in<sup>8</sup>. The main idea behind the quantum mechanical derivation of the equivalent photon approximation is the fact that photons with a small virtuality  $q^2$  are dominant due to the singular behavior of the photon propagator. The cross section for virtual photons can then be replaced by the one for real photons and the cross section is written as

$$\sigma(ee \rightarrow eeX) = \int \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} n(\omega_1) n(\omega_2) \sigma(\gamma\gamma \rightarrow X), \quad (1)$$

where  $n(\omega)$  is the equivalent photon number in the photon-photon case.

More recently photon-photon physics was also investigated for  $ep$  collisions or  $eA$  collisions at HERA<sup>9</sup>. In ion-ion collisions there are current measurements at STAR and both CMS and ALICE have plans to measure photon-photon and photon-hadron processes as well. In contrast to the point-like structure-less electrons with a small charge and only electromagnetic interaction the situation with ions is more complex. A number of additional processes, which occur in the photon-photon case, are shown in Fig. 2. Similar diagrams exist as well in the photon-ion case. Let us address the different processes in more detail.

The finite size of the ions can be taken care of by using the elastic form factor. The inelastic excitation will contribute also to the photon spectrum. This was studied for the excitation of the most important excited state, the giant dipole resonance (GDR), in<sup>10</sup>. It was found that this is only a rather small contribution, whereas, e.g., the  $p$ - $\Delta$  transition gives an additional effect of about 10%. At even higher  $q^2$  one also has incoherent contributions. For the proton it was found that the photon emission from individual quarks gives a contribution that is larger than the elastic one<sup>11,12</sup>. In the heavy ion case such a contribution is expected to be much smaller for two reasons: First the charge of the “partons” (quarks for the proton, protons for the heavy ion) is small compared to the total charge for ions, whereas they are of the same order for the proton. Therefore the loss of the coherence enhancement will be larger here. In addition the exclusion of the initial/final state interaction (see below) will remove a large part of this contribution. The virtuality of the photon in the inelastic case needs to be rather large,  $q^2 \gg 1/R^2$ ; their range will therefore be rather small. On the other hand the exclusion of the initial/final state interaction eliminates most of the small impact parameters.

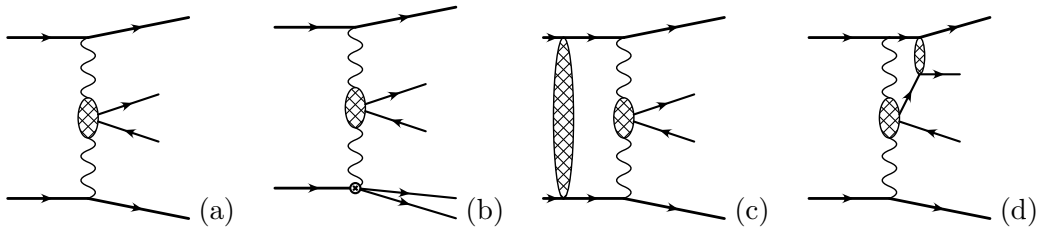


Figure 2: Due to the more complex nature of the ions corrections occur in photon-photon processes in the heavy ion case. Besides the elastic photon emission, which is governed by the elastic form factor (a), the photon can also be emitted inelastically (b). The two ions of course do interact with each other either via the Coulomb interaction or due to their nuclear interaction for small impact parameter (c) (“initial state interaction”). Finally if a hadronic final state is produced it can again interact with one of the ions (d) (“final state interaction”).

The exclusion of the nuclear interaction at small impact parameter (see Fig. 2(c)) is not easy to do in the plane wave approach. Of course photon-photon processes will occur in this case as well but they are completely dominated by the particles produced in the nuclear interactions; therefore these collisions are not useful and one should exclude them also from luminosity calculations. A possible approach for the inclusion of the strong interaction is a calculation starting from the eikonal approximation, see also the discussion in<sup>13</sup>. The strong Coulomb interaction between the two ions, characterized by the large Sommerfeld parameter  $\eta \approx Z_1 Z_2 \alpha \gg 1$ , on the other hand allows to use the semiclassical approximation, see also<sup>13</sup>. At high energies the heavy ions move essentially along a straight line and it is possible to define an impact parameter dependent photon number. In the case of a point-like particle this number is well known to be

$$N(\omega, b) = \frac{Z^2 \alpha}{\pi^2 b^2} \left( \frac{c}{v} \right)^2 x^2 \left[ K_1^2(x) + \frac{1}{\gamma^2} K_0^2(x) \right], \quad x = \frac{\omega b}{\gamma}. \quad (2)$$

In such a semiclassical pictures the exclusion of the nuclear interaction can be done by excluding all impact parameters with  $b < R_1 + R_2$ . Alternatively one can instead introduce the probability for no hadronic interaction given, e.g., from Glauber theory and get the photon-photon flux by integrating the product of the equivalent photon numbers and this probability over all impact parameter. In general the difference between this two approaches is not very large.

Another type of “initial state interaction” are additional electromagnetic processes occurring together with the photon-photon or photon-hadron interaction (Fig. 3). In the semiclassical approximation this process will be described by the product of the individual probabilities, which is discussed also in<sup>13</sup>. The main contribution is expected to be from the GDR, but excitation to higher energies contribute as well. For the heaviest ion the probability for GDR excitation is rather large and approaches one. Therefore the excitation of higher states (DGDR), see<sup>14</sup>, need to be taken into account. It is found that about 95% of all Pb-Pb collisions are accompanied by such an excitation, but only about 1% of the Ca-Ca collisions. As the equivalent photon number  $N(\omega, b)$  falls off as  $1/b^2$ , a very convenient parameterization of this probability is

$$P(A^*, b) \approx 1 - \exp \left[ -(\text{const}/b)^2 \right] \quad (3)$$

The importance of such an effect depends on the trigger condition used in the experiment. If one triggers for no breakup of the ions at all, one would loose this part of the luminosity. On the other hand this effect is currently used at RHIC to trigger for peripheral collisions by looking for the neutrons coming from the decay of the GDR in both ions, which are measures in both ZDCs<sup>15</sup>.

Due to their much larger ion luminosity, it was found that medium heavy ions (like Ca-Ca) give in the end a much larger total luminosity (given by the product of ion luminosity and photon-photon luminosity). Therefore medium heavy ion beams are often more preferable. In

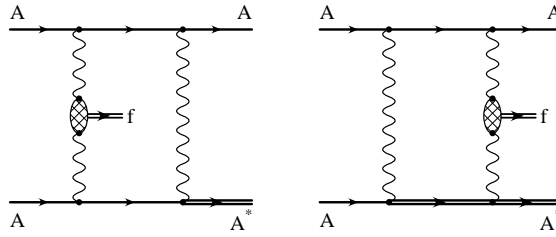


Figure 3: The strong fields allow for the excitation of the ions in addition to the photon-photon process. For the heaviest ion the probability for this is rather large. The semiclassical approximation allows for a very easy description of these processes in terms of the products of the individual probabilities.

addition the ions have a smaller size and therefore the maximum photon energy  $\omega_{max} = \gamma/R$  will be larger in this case as well.

Apart from the proton-proton case (see also <sup>16</sup>) it will probably be impossible to measure the scattered ions, therefore only “untagged events” will be possible. These are characterized by a small total transverse momentum of the final state smaller than about  $2/R$ . In terms of the rapidity the photon luminosity peaks at  $Y = 0$  and has a range of about

$$Y_{max} \approx \ln 2\omega_{max}/M_{\gamma\gamma}. \quad (4)$$

### 3 Physics Potential in $\gamma\gamma$ and $\gamma A$ collisions

The available invariant mass range in the photon-photon case is given by the maximum photon energy  $\omega_{max} \approx \gamma/R$  and therefore the maximum invariant mass is  $2\gamma/R$  <sup>a</sup>. The invariant mass range for RHIC extends up to a few GeV and therefore makes this of interest for meson spectroscopy in this range. The luminosities are comparable to what has been available at LEP. Both meson production and meson pair production can be studied. Detailed studies of these processes have been done by the “Ultrapерipheral Collision Group” at STAR, see also <sup>17,18</sup>. The suppression of the two-photon production of a meson would also be an indication for its gluonic nature, making it possible to look, for example, for glueballs.

For the LHC the invariant mass range goes up to a few 100 GeV and therefore extends the possibilities of LEP both in mass range and in luminosity. Meson spectroscopy is possible there, especially for mesons with  $b$  and  $c$  quarks, see Fig. 4(a). In addition the total cross section  $\gamma\gamma \rightarrow \text{hadrons}$  has been measured at LEP. Deviations from the Pomeron universality have been found there. Photon-Photon processes at the LHC have the potential to look for this process at higher energies. In addition it is worthwhile to study whether the detectors there will be able to detect also in the more forward direction, where most of the events will occur and which up to now are undetectable and need to be modeled.

The possibility to find new particles at the LHC has attracted some interest in the past. Especially Higgs boson production and the search for supersymmetric particles have been studied in detail. Unfortunately the production rates (for ion-ion collisions) seem to be rather small (few per year) in these cases and for realistic parameters.

Photon-ion collisions are possible over a large range of energies. Please remember that the maximum photon energy in this case is given by  $\omega_{max} = \gamma_{ion}/R$  with the Lorentz factor  $\gamma_{ion} = 2\gamma_{coll}^2 - 1$  in the rest frame of one of the ions. Therefore the maximum photon energy is about 300 GeV for RHIC and about 500 TeV for the LHC. The total photon-ion cross section is dominated by the nuclear excitation mainly of the GDR (the so called “Weizsäcker-Williams

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<sup>a</sup>Of course these “maximum energies” should not be treated as absolute maxima. Photon with higher energies do occur but their spectrum decreases exponentially.

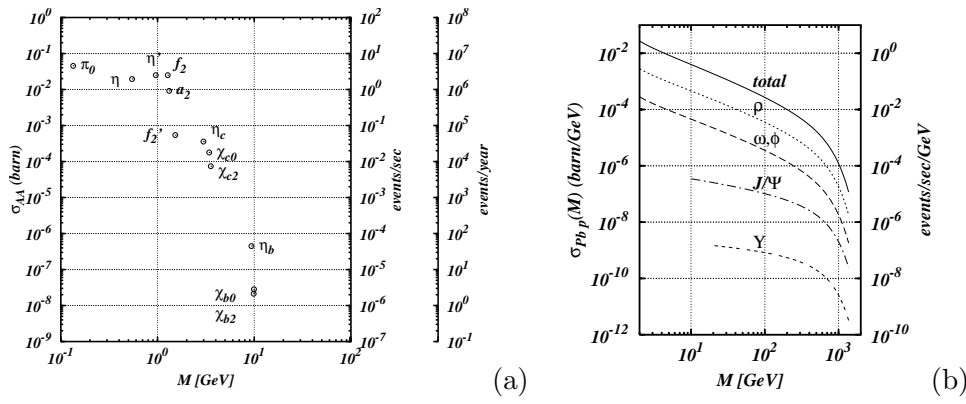


Figure 4: The cross sections and rates for different mesons in  $\gamma\gamma$  collisions are shown for meson production in Pb-Pb collisions at the LHC in (a). Figure (b) shows the same properties for the diffractive vector meson production as well as the total hadronic cross section for a hypothetical collision of Pb on protons for LHC energies. The results for Pb-Pb collisions can be obtained by a multiplication of a factor describing the coherent (or incoherent) production. For the rates a ion-ion luminosity of  $10^{26} \text{ cm}^{-2} \text{ s}^{-1}$  was assumed.

process”) at the lowest energies. This is an important loss process as the excited nuclei decay mostly via neutron emission. But it is also used at RHIC as a luminosity monitor, see <sup>19</sup>. Going to higher energies coherent diffractive processes are of interest. At the LHC the coherent diffractive vector meson production on a nucleus up to the Upsilon can be studied. The expected rates for the hypothetical collisions of photons from Pb on a proton are shown in Fig. 4(b). For the heavy ion case they need of course to be multiplied by a factor depending on the details of the coherent production on the nuclei. In this way one might be able to see the transition from the “soft” to the “hard” Pomeron with increasing meson mass. Looking for vector mesons with a larger transverse momenta one will also be able to study the inelastic vector meson production. Again Fig. 4(b) will be helpful to estimate the rates. Finally it has been proposed to study the photon-gluon fusion process and with it to be able to measure the in-medium gluon distribution function. Such a measurement would be very interesting as some models predict a saturation of the gluon density at small Bjorken  $x$ .

#### 4 Lepton Pair Production and QED of Strong Fields

An interesting subject in itself is the production of light lepton pairs. Electron-positron pair production (and to some extent also muon pair production) plays a special role in peripheral heavy ion collisions. Due to their small mass they are produced quite easily. It is interesting to note that the work of Landau and Lifschitz in 1934 <sup>20</sup>, followed by Racah 1937 <sup>21</sup>, are probably the first calculation of an electromagnetic process in relativistic ion collisions. The cross section for  $e^+e^-$  pair production is quite large. Using, e.g., the formula derived by Racah <sup>21</sup>

$$\sigma = \frac{Z^4 \alpha^4}{\pi m^2} \frac{28}{27} \left[ \ln^3 \gamma_{ion}^2 - 2.19 \ln^2 \gamma_{ion}^2 + \dots \right], \quad (5)$$

one finds cross sections of about 30kbarn for Au-Au collisions at RHIC and 200kbarn for Pb-Pb collisions at LHC, leading, e.g., to  $10^7$  pairs/sec produced in Pb-Pb collisions at LHC. This cross section increases with  $\ln^3 \gamma_{ion}^2$ , which is rather fast and will eventually lead to a violation of the Froissart bound. Therefore new phenomena will occur at high energies.

Most electrons and positrons are produced with rather modest energies (of the order of several  $m_e$ ) and into the very forward direction. Therefore they will remain unobserved. Still the cross section for large angles and large energies is quite sizable. Finally we want to point out that the equivalent photon approximation can only be used with great care for  $e^+e^-$  pair

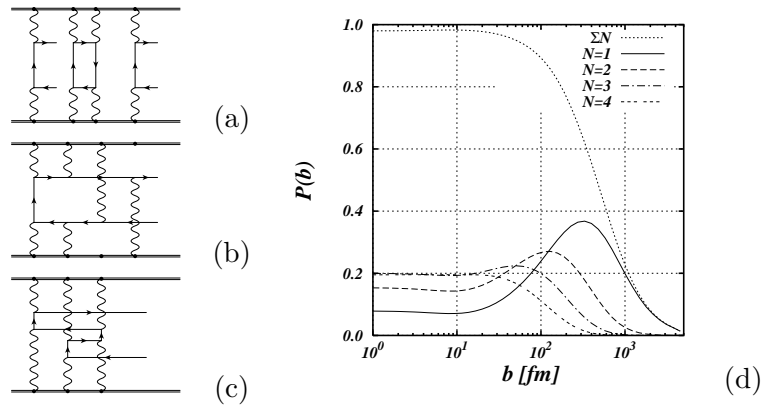


Figure 5: The electromagnetic fields in relativistic heavy ion collisions are so strong that higher order processes will occur. Multiple pair production in a single collision is possible as shown in (a). Coulomb corrections lead to the distortion of the wave functions of electron and positrons (b). Finally due to the strong fields also multiple-particle processes can occur (c). The impact parameter dependent probability for the production of up to four pairs in lowest order is shown in (d) for Pb-Pb collisions at the LHC.

production. This is due to the smallness of  $m_e$  compared to  $1/R$ . The total cross section can be calculated within EPA by using as cutoff parameter  $m_e$  instead of  $1/R$  or restricting the impact parameters to be larger than  $\lambda_c = 1/m_e \approx 400\text{fm}$ . Calculations using only a restricted part of the phase space or for small impact parameter on the other hand need to be done either with a more refined analysis of this cutoff parameter or using a full calculation.

#### 4.1 Multiple Pair Production and Coulomb Corrections

Electron-positron pair production has attracted some interest in recent years due to the observation that the impact parameter dependent probability calculated in perturbation theory can exceed unity already for RHIC and for impact parameters up to the Compton wavelength of the electron ( $\lambda_c \approx 400\text{fm}$ )<sup>22</sup>. That the pair production cross section rises too fast with collision energy was already observed by Heitler<sup>23</sup>, but was thought to be an “academic problem” by him. In a series of papers starting from<sup>22</sup> it was found that the probability larger than one means that more than one pair will be produced on the average within a single collisions (Fig. 5(a)).

Neglecting the antisymmetrization of the final state (and therefore treating the pair as a “quasiboson”) the probability  $P(N, b)$  for the  $N$  pair production was found to follow a Poisson distribution<sup>24,25</sup>

$$P(N, b) = \frac{P^N(b)}{N!} \exp(-P(b)) \quad (6)$$

with  $P(b)$  the probability for a single lepton line.  $P(b)$ , which is the probability calculated, e.g., in lowest order perturbation theory, is naturally interpreted as the average multiplicity of pairs

$$\langle N \rangle(b) = P(b). \quad (7)$$

Corrections to the Poisson distribution, as well as multiple-particle effects, have been discussed in<sup>24</sup>, where also the earlier work is discussed.

The impact parameter dependent probability was calculated in lowest order<sup>26</sup> and it was found that on the average three to four pairs will be produced for small impact parameter. Cross section for the production of up to five pairs have been given as well<sup>25</sup> (see Fig. 5(d)). The results for impact parameter of the order of a few  $\lambda_c$  have been recently confirmed<sup>27</sup>.

As the fields are rather strong (the effective coupling constant  $Z\alpha = 0.6$  is not small) one might doubt the results of the lowest order calculations, expecting that higher order processes

(on the single pair production process) should be important. Two different types of higher order corrections can be loosely distinguished<sup>28</sup>: The distortion of the electron and positron wavefunctions after their production are normally subsumed under the term “Coulomb corrections”, see also Fig. 5(b). A second class deals with the fact that more than one pair can be present at an intermediate stage before the pairs annihilate crosswise (Fig. 5(c)). Such “multiple-particle” processes<sup>24</sup> are unique to the situation of heavy ion collisions with two strong fields. A calculation for small impact parameter<sup>24</sup> found them to be rather small.

A classical result for Coulomb corrections in the pair production cross section are the Bethe-Maximon corrections<sup>29,30</sup>, describing the production of an electron-positron pair through a single (real) photon interacting with a strong Coulomb field<sup>b</sup>. In the Bethe-Heitler formula

$$\sigma = \frac{28}{9} \frac{(Z\alpha)^2}{m_e^2} \left[ \ln \frac{2\omega}{m_e} - \frac{109}{42} - f(Z\alpha) \right], \quad (8)$$

the Coulomb corrections are contained in the correction term

$$f(Z\alpha) = (Z\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + (Z\alpha)^2)} = \gamma + \text{Re}\psi(1 + iZ\alpha) \quad (9)$$

with the Euler constant  $\gamma \approx 0.57721$  and  $\psi$  the Psi (or Digamma) function.

Using an analysis of the different orders in  $\ln \gamma$  of the contribution with either *one* or *multiple* photon emissions from each ion, the Coulomb corrections to the total *cross section* are calculated in<sup>31</sup>. Rather large reductions compared to the Born cross sections (about 25% and 14% for RHIC and LHC) are given. Such an analysis is unfortunately not helpful for the calculation of multiple pair production as this requires the exchange of *multiple* photons from each ion.

A different approach was proposed by a number of authors<sup>32,33,34</sup> making use of the form of the interaction in the high energy limit  $\gamma \rightarrow \infty$ . Using retarded boundary conditions it is found that only a certain class of diagrams is dominant in the high energy limit. The sum to all orders can be related to the one in lowest order approximation by a replacement of the photon propagator

$$\frac{1}{q^2} \rightarrow \frac{1}{(q^2)^{1-iZ\alpha}} \quad (10)$$

A calculation of the impact parameter dependent probability was done in<sup>35</sup>, where it was found that the Coulomb corrections lead to a substantial reduction of the probabilities at small impact parameter and therefore also of the multiple-pair production cross sections.

It was already pointed out in<sup>33,36</sup> that this high energy limit leads to the interesting result that the cross section (but not  $P(b)$  itself) including all higher orders is identical to that in lowest order. This is of course in disagreement to the Bethe-Maximon corrections as discussed above. Different solutions for this have been proposed in the mean time (for a discussion see<sup>4</sup>). That a too simplified use of the eikonal approximation will lead to a result in disagreement with Bethe-Maximon theory was already pointed out in<sup>45</sup>. Using a more refined analysis<sup>37</sup> the Bethe-Maximon corrections were found to be present here as well. Still the Coulomb corrections for the multiple pair production process have not been calculated up to now.

#### 4.2 Bound-Free Pair Production

The electron from the produced pair has a tiny probability to be produced not as a free particle but into the bound state of one of the ions

$$A + A \rightarrow A + (Ae^-)_{K,L,\dots} + e^+ \quad (11)$$

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<sup>b</sup>This coincides with the limit  $Z_1 \rightarrow 0$  in the heavy ion case.

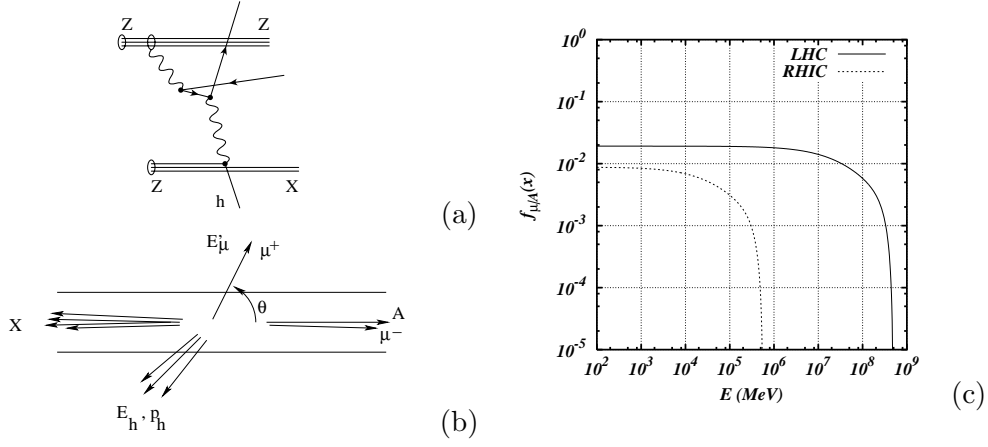


Figure 6: Processes with one muon emitted under a large angle and a jet in the opposite direction can be thought of to occur from the deep inelastically scattering of an equivalent muon (a). They are characterized by one muon scattered into one direction and a jet in the opposite one (b). The spectrum of the muons as a function of their energy is shown for both Pb-Pb collisions at LHC and Au-Au collisions at RHIC (c).

Even though this probability is small in combination with the large cross section for the pair production makes for a cross section of the order of 100 barn. As the  $Z/A$  ration changes to  $(Z - 1)/A$  in this process, the ion is lost from the beam. Together with the electromagnetic excitation of the ions themselves this is the dominant loss process. But whereas the excited ions, which subsequently decay mostly by neutron emissions have a large momentum spread, the momentum transfer coming from the electron capture is small and a very narrow  $\text{Pb}^{81+}$  beam emerges after the interaction region. Recently it was shown<sup>38</sup> that this narrow beam of ions will hit the wall hundreds of meters after the interaction region in a very narrow spot depositing an energy of the order of tens of watts in a very small spot and the maximum beam luminosity of the Pb beam at the LHC is limited due to the magnet cooling in this region.

Recently a full calculation of the bound-free pair production in first order in the target interaction was made not only for production into the  $1s$ -state but also to higher  $s$  and  $p$  states<sup>39</sup>. The results were also compared with other calculations up to now. In general a good agreement was found, therefore one expects that the size of this process seems to be well under control.

#### 4.3 Equivalent Leptons: Heavy Ions as a source of high energetic leptons

The process of lepton (in the following mainly muon) production with a large transverse momentum of one of the muons can be best described within the picture of the “equivalent lepton approximation”<sup>40,41</sup>. The photon emitted from one of the ions contains muons as partons, which can scatter deep inelastically with the other ion (see Fig. 6(a)). Using this approach the spectrum of the equivalent muons (or electrons) can be written as

$$f_{\mu|Z}(x) = \int_x^{1/(m_A R)} du f_{\gamma|Z}(u) f_{\mu|\gamma}(x/u). \quad (12)$$

With this the cross section can be calculated, see Eq. (144) of<sup>4</sup>. The equivalent muon spectrum for RHIC and LHC are given in Fig. 6(c).

The deep inelastic scattering process will give information about the quark distribution function within the nucleus. In contrast to a lepton beam, which is mono-energetic, the muon here has a continuous spectrum. Nevertheless by measuring the scattered muon and the energy and longitudinal momentum of the jet going into the opposite direction (see Fig. 6(b)) one is



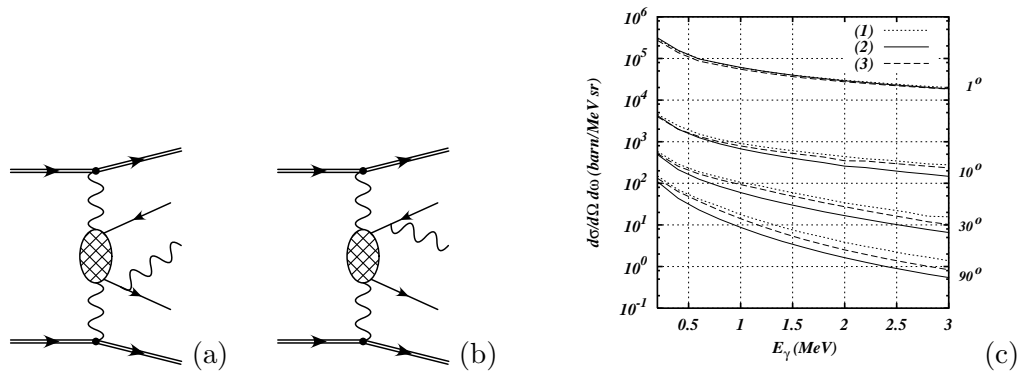


Figure 7: The emission of photons from the created electrons and positrons is the main source of bremsstrahlung at “high” photon energies and large angles. In the IR limit diagrams (a) and (b) give the main contribution. The result of a calculation using three different approximation schemes are shown for the conditions of Pb-Pb collisions at the LHC in (c).

able to reconstruct the energy of the initial muon. Of course these cross section are small. Nevertheless it is worthwhile to study how these processes can be used and work on this is in progress.

#### 4.4 Bremsstrahlung from produced pairs: The “glowing of heavy ion collisions”

The emission of bremsstrahlung photons from the ions itself is highly suppressed due to their large mass<sup>42</sup>. On the other hand the pair production cross section is rather large and due to their small mass bremsstrahlung emission is possible easily. It is well known that in the case of lepton colliders this process can become the dominant source for photons at large angles<sup>43</sup>. A calculation of the bremsstrahlung spectrum was done using different approximations<sup>44</sup>. The results for different energies and angles are shown in Fig. 7(c). It is an open question whether these processes could be used, e.g., in order to “see” the interaction vertex of the two ion bunches directly by looking at these photons.

## 5 Summary

The strong electromagnetic fields can be used for both photon-photon and photon-ion processes. These processes are measurable in very peripheral collisions, as only the long range electromagnetic interaction is present in this case. They are interesting processes to be studied at both RHIC and LHC. The theoretical description of these processes is well understood and corrections to the dominant coherent photon emission have been studied already. Electron-positron pair production plays a special role in these collisions. Besides the theoretical interest connected to the strong field effects, they are also of practical importance as a possible loss process or even as a possibility to measure the parton distributions inside the ions.

As can also be seen from the other contributions to this workshop, the beautiful results of the STAR detector at RHIC have shown that we have definitely left the regime of theoreticians dreams and hopes and are entering the aera of experimental realities and results.

## Acknowledgments

The author would like to thank Sebastian White and Bill Marciano for the organization of this workshop and the possibility to participate. The wonderful atmosphere has lead to a number of most interesting discussions, which will be surely very fruitful for the future of this kind of physics.

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